On some recent advances for under-controlled coupled PDE's and applications to insensitizing and simultaneous control

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Abstract. Many applications in control theory are concerned with the controllability of coupled systems of PDE's. A topic which has recently emerged in this field is concerned with controllability issues for under-controlled coupled systems, characterized by the fact that the number of controls is strictly less than the number of equations. Such systems appear naturally when one wants to build robust controls for scalar equations, that is controls which are robust to small unknown perturbations of the initial data or when one wants to control simultaneously devices in parallel. They arise more generally in applications, whenever cost constraints or practical realizations, may require that the controls are activated only for some of the physical variables or for some of the equations.

We shall present some recent advances, based on a general macroscopic method: the multi-levels energy method, for the locally distributed and boundary control/observability of certain types of structured coupled hyperbolic systems, namely here cascade system. This method mixes different levels of energies of the components of the state vector according to the fact that they are observed or unobserved components. Further time invariance properties of these systems such as time-translation invariance, reversibility and conservation of some energies, play an important role. It allows us to deal with controllability/observability issues for such structured under-controlled coupled systems, in some "unified way". A further important concern is to obtain results for geometric situations in which the control and coupling regions do not intersect. In particular, we give a necessary and sufficient condition for the observability of the dual cascade system of two equations by a single observation. This condition, roughly speaking states that both the control and coupling region should satisfy the Geometric Control Condition. We further generalize this result to bi-diagonal cascade systems of n equations.These results are valid for sufficiently large control time. We then give applications to the existence of insensitizing and simultaneous locally distributed or boundary controls of scalar equations in a multi-dimensional framework and discuss some perspectives.

References

[1] F. Alabau-Boussouira, *Indirect boundary observability of a weakly coupled wave system,* C. R. Acad. Sci. Paris, t. 333, Série I (2001), 645–650.

[2] F. Alabau-Boussouira, *A two-level energy method for indirect boundary observability and controllability of weakly coupled hyperbolic systems,* SIAM J. Control Opt., 42 (2003), 871–906.

[3] F. Alabau-Boussouira and M. Léautaud, *Indirect controllability of locally coupled systems under geometric conditions,* C. R. Acad. Sci. Paris, t. 349, Série I (2011), 395–400.

[4] F. Alabau-Boussouira and M. Léautaud, *Indirect controllability of locally coupled wave-type systems and applications,* J. de Mathématiques Pures et Appliquées, 99 (2013), 544–576.

[5] F. Alabau-Boussouira, *Controllability of cascade coupled systems of multi-dimensional evolution PDE's by a reduced number of controls,* C. R. Acad. Sci. Paris, t. 350, Série I (2012), 577–582.

[6] F. Alabau-Boussouira, *Insensitizing controls for the scalar wave equation and exact controllability of 2-coupled cascade systems of PDE's by a single control,* Mathematics of Control, Signals, and Systems (2013).

[7] F. Alabau-Boussouira, A hierarchic multi-level energy method for the control of bi-diagonal and mixed n-coupled cascade systems of PDE's by a reduced number of controls. Advances in Differential Equations, 18, 1005–1072 (2013).

[8] R. Dáger, *Insensitizing controls for the 1-D wave equation,* SIAM J. Control Optim., 45 (2006), 1758–1768.

[9] B. Dehman, J. Le Rousseau, and M. Léautaud, *Controllability of two coupled wave equations on a compact manifold conditions,* To appear in Arch for Rational Mech. Anal.

[10] J.L. Lions, *Remarques préliminaires sur le contrôle des systèmes à données incomplètes,* Actas del Congreso de Ecuaciones Diferenciales y Aplicaciones (CEDYA), Universidad de Málaga, (1989), 43–54.

[11] L. Rosier and L. de Teresa, *Exact controllability of a cascade system of conservative equations,* C. R. Acad. Sci. Paris, t. 349, Série I (2011), 291–296.

[12] L. Tebou, *Some results on the controllability of coupled semilinear wave equations: the desensitizing control case,* SIAM J. Control Optim., 49 (2011), 1221–1238.