

The geometry of the optimal control of a linear three-spin system with Ising coupling

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November 14, 2013

Abstract

The objective of this talk is to introduce the geometric framework to analyze the time optimal control of a linear three-spin system with Ising couplings. The problem can be lifted as a SR-invariant problem on $SO(3)$ and the integrability properties of the extremal flows are deduced with parameterizations of the solutions using elliptic functions. Direct computations on the homogenous space $S^2 = SO(3)/SO(2)$ lead to analyze a one parameter family of Riemannian metrics with a Grushin singularity at the equator and which are Liouville integrable with additional Casimir type first integral given by the Hamiltonian of the usual round metric on the sphere. This related the computations of the conjugate and cut loci to 2D-Liouville metrics on S^2 to recent works generalizing the case of ellipsoids.