Numerical controllability of the wave equation using finite elements for a space-time discretization

Nicolae Cîndea*

Short talk application for MCT2013

The aim of this talk is to present some recent results [1, 2] concerning the numerical approximation of exact boundary controls for the wave equation with a potential. The goal is to compute approximations of controls that drive the solution from a prescribed initial state to zero at a large enough controllability time. In this purpose we use primal and dual methods coupled with Carleman estimates and with an augmented Lagrangian method respectively.

The specificity of these two methods is that the boundary control and the controlled solution are obtained in terms of a new variable – the solution of a fourth-order elliptic problem defined in the space-time domain. For the primal method we prove that, for some specific weights determined by the global Carleman inequalities for the wave equation, this problem is well-posed. For the dual method, we prove a inf-sup condition which is sufficient for the existence and uniqueness of solutions for the corresponding elliptic problem.

In both cases, in the framework of the finite element method, we introduce a family of finite-dimensional approximate control problems and we prove a strong convergence result. Numerical experiments confirm the analysis.

References

cfcm [1] N. Cîndea, E. Fernández-Cara, A. Münch, *Numerical controllability of the wave equation through primal method and Carleman estimates*. ESAIM COCV. Accepted 07 Jan 2013. http://hal.archives-ouvertes.fr/hal-00668951.

cm [2] N. Cîndea, A. Münch, *Numerical controllability of the wave equation through dual methods and Lagrange multipliers*. Work in progress.

*Laboratoire de Mathématiques, Université Blaise Pascal (Clermont-Ferand 2), UMR CNRS 6620, Campus de Cézeaux, 63177, Aubière, France. E-mail: nicolae.cindea@math.univ-bpclermont.fr