Normal parabolic equations: structure of dynamical flow and nonlocal feedback stabilization.

A. V. Fursikov,
Moscow State University
Departmant of Mechanics and Mathematics

As well-known, existence of weak solution for Navier-Stokes system is obtained with help of energy estimate. Absents of such bound in phase space H^1 in 3D case is serious obstacle to get nonlocal existence of smooth solutions.

Semilinear parabolic equation is called equation of normal type if its nonlinear term B satisfies the condition: $\forall v \in H^1$ $B(v)$ is collinear to v. Since the property $B(v) \perp v$ implies analog of energy estimate in H^1, equation of normal type does not satisfies energy estimate „in the most degree".

For 3D Helmholtz equations (that are analog of Navier-Stokes system where the curl of fluid velocity is given as unknown function) we derive corresponding normal parabolic equations (NPE). This means by definition that nonlinear term $B(v)$ of constructed NPE is orthogonal projection of nonlinear term $B_H(v)$ for Helmholtz system on the ray generated by v. It is turn out that there exists explicit formula for solution to NPE with periodic boundary conditions. This formula is used to study the structure of dynamical flow corresponding to NPE. We prove that its phase space V can be decomposed on the set of stability M_- (solutions with initial condition $\omega_0 \in M_-$ tends to zero with prescribed rate $e^{-\alpha t}$ as time $t \to \infty$), set of explosions M_+ (solutions with initial condition $\omega_0 \in M_+$ blow up during finite time), and set of growing M_G (norm of solutions with initial condition $\omega_0 \in M_G$ tends to infinity as time $t \to \infty$. The exact description of all these sets is given.

We hope to use obtained results to understand better questions connected with nonlocal solvability of 3D Helmholtz system.

All aforementioned results are true in the case of NPE corresponding to Burgers equation. Besides, in this case we construct nonlocal feedback stabilization of NPE solution by either starting or impulse or distributed control. In all cases we assume that this control is supported in an arbitrary fixed subdomain of the spatial domain.