Model Predictive Regulation

Cesar O. Aguilar
Department of Mathematics
California State University, Bakersfield
and
Arthur J. Krener
Department of Mathematics
University of California, Davis

Abstract;

Frequently one wishes to stabilize a process to an operating point without using too much control effort. There may be many ways of doing this so often this is formulated as an optimal control problem the solution of which is a stabilizing feedback. The solution to the optimal control problem is obtained by solving the Hamilton-Jacobi-Bellman (HJB) partial differential equation or its discrete time analog, the Dynamic Programing Equation (DPE). But these equations are notoriously difficult to solve even in moderate state dimensions. Therefore there has been a trend in the process control and other industries to use Model Predictive Control (MPC). Instead of finding the optimal solution for all possible initial conditions, MPC finds the optimal solution from the current state , utilizes the optimal control over one time step and then repeats the process at the next state. The criterion to be minimized is a running cost over a finite horizon plus a terminal cost chosen to ensure asymptotic stability.

But suppose one does not wish to stabilize to an operating point but rather to track a reference trajectory or reject a disturbance generated by an exosystem. Then in addition to solving for HJB or DPE one has to solve the Francis-Byrnes-Isidori (FBI) partial differential equation or its discrete time analog. These are also very difficult to solve even in moderate state dimensions. Model Predictive Regulation (MPR) is a way to solve both HJB and FBI (or their discrete time analogs) in one step using MPC methodology. The key is to recast the tracking problem as an optimal control problem where the running cost is zero when tracking (or disturbance rejection) is being achieved. In continuous time the desired running cost is a convex function of the output to be regulated and its time derivatives up to its relative degree. In discrete time the desired running cost is a convex function of the output to be regulated and its time advances up to its relative degree. The resulting running cost should be convex in the state of the plant and exosystem and strictly convex in the control. Then in effect the MPC algorithm solves both the HJB and the FBI equations simultaneously.