Mathematical Control in Trieste : Contributed talk

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"Simultaneous global exact controllability of an arbitrary number of 1D bilinear Schrödinger equations"

This talk deals with simultaneous global exact controllability of an arbitrary (finite) number of 1D bilinear Schrödinger equations

$$\begin{cases} i\partial_t \psi^j = \big(-\partial^2_{xx} + V(x)\big)\psi^j - u(t)\mu(x)\psi^j, & (t,x) \in (0,T)\times(0,1),\, j \in \{1,\dots,N\}, \\ \psi^j(t,0) = \psi^j(t,1) = 0, & t \in (0,T),\, j \in \{1,\dots,N\}. \end{cases} \tag{1}$$

This is a joint work with Vahagn Nersesyan [3].

In this setting we have N identical equations controlled by a single control u. This control is the amplitude of the external field applied on the particles. Our controllability result holds for an arbitrary potential V with generic assumptions on the dipole moment μ.

From the local exact controllability around the ground state in $H^3_{(0)}$ obtained by K. Beauchard and C. Laurent [1] and the global approximate controllability towards the ground state obtained by V. Nersesyan [4], global exact controllability was known for the case of a single equation with $V = 0$.

Our proof combines three steps. Using Lyapunov-like ideas we prove global approximate controllability towards vectors that are finite sums of eigenvectors.

The simultaneous local exact controllability is obtained thanks to the return method introduced by J.-M. Coron. As in [2] the reference trajectory is designed using partial control results. The controllability of the linearized system around this reference trajectory relies on suitable Riesz basis properties.

Simultaneous global exact controllability is then obtained by connectedness and compactness arguments.

Finally, this result is extended to arbitrary potentials using translated controls.

References

[1] K. Beauchard and C. Laurent. Local controllability of 1D linear and nonlinear Schrödinger equations with bilinear control. *J. Math. Pures Appl. (9)*, 94(5):520–554, 2010.

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[3] M. Morancey and V. Nersesyan. Simultaneous global exact controllability of an arbitrary number of 1D bilinear Schrödinger equations. preprint, arXiv:1306.5851, 2013.

[4] V. Nersesyan. Global approximate controllability for Schrödinger equation in higher Sobolev norms and applications. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 27(3):901–915, 2010.

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