Control problems in models of the interaction between a population and individuals

Nikolay Pogodaev

The talk is devoted to control problems arising in models of the interaction between a population and some individuals. Several specific situations can be considered: a shepherd dog leading a herd of sheep, several police officers trying to hold a crowd of protesters, a predator attacking a group of preys etc. There are two ways to describe such situations: one leads to a controlled differential inclusion, another to a controlled conservation law.

More precisely, let a map $\xi = \xi(t)$ describe the movement of the k individuals (e.g. the k shepherd dogs) in $\mathbb{R}^2$. Assume that the population is characterized by the region $K_\xi(t) \subset \mathbb{R}^2$ it occupies at time t. Then, $K_\xi(t)$ is the reachable set of the differential inclusion

$$\dot{x} \in v\left(x, \xi(t)\right) + B(0, c), \tag{1}$$

where $B(0, c)$ is the closed ball in $\mathbb{R}^2$ centered at 0 with radius c, c being the (maximal) wandering speed of the population's members. The vector field v is the drift speed due to the interaction between the k individuals and the population.

On the other hand, let the population be characterized by a density function $\rho = \rho(t, x)$ (the number of population's members per unit area). Then, ρ evolves according to the law of mass conservation

$$\rho_t + \mathrm{div}_x(\rho V) = 0. \tag{2}$$

Here $V = v(x, \xi) + v_{\mathrm{int}}(\rho)$ is the velocity of a population's member located at x.

In both models the function ξ can be considered as a strategy of the individuals, which they choose in order to achieve some goal. Thus, several control problems naturally arise: *the confinement problem* (find a strategy of the individuals that allows to hold the population inside a given set within a given period of time), *the steering problem* (find a strategy that allows to steer the population into a given set by a given time), *the optimal control problem* (find a strategy that minimizes some cost functional).

In the talk the recent results [1, 2] concerning these control problems for model (1) will be presented. Some connections between models (1) and (2) will be discussed as well.

References

[1] R. M. Colombo and N. Pogodaev. Confinement strategies in a model for the interaction between individuals and a continuum. *SIAM J. Appl. Dyn. Syst.*, 11(2):741–770, 2012.

[2] R. M. Colombo and N. Pogodaev. On the control of moving sets: Positive and negative confinement results. *SIAM J. Control Optim.*, 51(1):380–401, 2013.