**Observer convergence from the Riemannian point of view**

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An observer whose state lives in a copy of the given system's space and which guarantees the property that a Riemannian distance between system and observer solutions is nonincreasing is such that that the Lie derivative of the Riemannian metric along the system vector field is negative in the space tangent to the output function level sets. Also, if the observer has an infinite gain margin then the level sets of the output function are geodesically convex.

Conversely, if these two properties are satisfied, then there exists an observer with an infinite gain margin.

The existence of a Riemannian metric satisfying these two properties is guaranteed for systems that are strongly differentially observable.

The existence of a Riemannian metric satisfying the inequality on its Lie is also guaranteed when the time-varying linear systems resulting from the linearization along solutions to the system satisfy an observability property. On the other hand, the fact that it makes the output level sets geodesically convex is equivalent to the existence of a reduced order observer.