Control and mixing for nonlinear equations

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We discuss the interconnection between controllability properties of a dynamical system and large-time asymptotics of trajectories for the associated stochastic system. We begin with a result on the finite-dimensional case which applies to differential equations on a smooth Riemannian manifold. It will be proved that the approximate controllability to a given point and local solid controllability imply the uniqueness of a stationary measure and exponential mixing in the total variation distance. We next turn to problems in infinite dimension and derive a sufficient condition (in terms of controllability properties) for the exponential mixing in the Kantorovich-Wasserstein distance. This result applies, for instance, to the 2D Navier-Stokes system driven by a random force acting on the boundary. We conclude with some open problems on controllability properties of the Navier-Stokes system, which would have interesting applications in the ergodic theory of the associated random flow.